In routine practical analysis one must consider the relationship between economic requirements and the use of assumed functions with greater numbers of undetermined parameters than there are element node point displacements. Normally, the objective of engineering analysis is to obtain an acceptable result at minimum cost. The use of "excess" undetermined parameters may not only complicate the formulation but will certainly add expense to the computation for a given network size. Thus, it is possible that the simplest element relationships will provide acceptable results with the coarsest network one would choose to use, or else, with a larger network, at less cost. Note the closeness of the values of the terms in the stiffness matrix of Ref. 1 for 5 and 7 term stress assumptions, respectively; equations for the former, however, can be simply and explicitly formulated whereas equations for the latter are not readily formulated and otherwise require significant matrix operations.

Finally, a more complete exploitation of matrix structural analysis methods involves their use in instability, vibration, thermal stress, and inelastic analyses, among others. Consequently, it would be desirable for examinations of concepts in the formulation of discrete element relationships to include consideration of the appropriate terms for these phenomena.

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Reply by Author to R. H. Gallagher

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THE author wishes to point out that Eq. (23) of Ref. 1 has been chosen to insure the boundary compatibility between neighboring elements. For example, for two neighboring rectangular elements (I) and (II) as shown in Fig. 1, the displacements $u(y)$ along the edge BC of both elements are given by the same function

$$u_{BC}(y) = [1 - (y/b)]u_B + (y/b)u_C \tag{1}$$

where u_B and u_C are the horizontal displacements at corners B and C, respectively. It is seen that the edge displacement $u_{BC}(y)$ of either element is not affected, for example, by a vertical displacement at A or F.

If the displacement functions given by Eqs. (3) of the foregoing comment is employed, two different displacement functions will be resulted. They are, on element (I),

$$u_{BC}(y) = \left(1 - \frac{y}{b}\right) u_B - \frac{y}{b} u_C + \frac{y}{2a}\left(1 - \frac{y}{b}\right) \times (v_B - v_B + v_C - v) \tag{2}$$

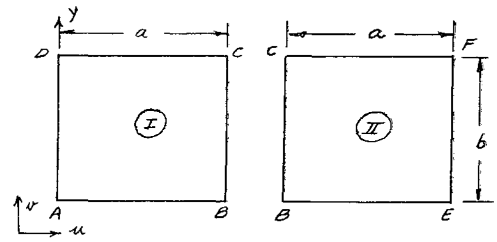

Fig. 1 Two neighboring rectangular elements.

and on element (II),

$$u_{BC}(y) = \left(1 - \frac{y}{b}\right) u_B + \frac{y}{b} u_C + \frac{y}{2a}\left(1 - \frac{y}{b}\right) \times (v_B - v_E + v_F - v_C) \tag{3}$$

Hence the two horizontal displacements will, in general, not be compatible because $(v_A - v_B - v_C - v_D)$ and $(v_B - v_E + v_F - v_C)$ are not correlated.

Thus, the author has a feeling that the numerical result given at the bottom of page 1335 of Ref. 1 are not identical to those which would be obtained by use of the equations formulated by Turner, et al., in Ref. 2.

The author is in full agreement with Gallagher that both accuracy and cost should be taken into consideration when a method is selected for a routine practical analysis, and hence there is a limit for the number of undetermined parameters to be used in formulating the stiffness matrix. The author, however, has been trying to find out whether there is basically a stiffness matrix that will yield results better than others when the same size networks are used.

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Comments on "Effect of Gas Composition on the Ablation Behavior of a Charring Material"

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A PAPER by Vojvodich and Pope[1] presented the results of an experimental investigation on the ablation behavior of a charring material. Chemical reactions were shown to occur between the material and an air environment, and the gas phase reactions were found to be more significant than predicted by the theory derived from the work of Cohen, Bromberg, and Lipkis and the work of Hartnett and Eckert.[2] A close examination of the experimental results reveals a possible discrepancy in the reported combustion effects. This observation follows from a thermochemical analysis of the ablative material and a discussion and analysis reported in Ref. 3. Additional clarification of the theory[2] and notation of the similarity to the theory of Lees[4] should enhance the interpretation of the experimental results.

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